Problem 11.8

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Problem 11.8 should be moved to become problem 10.13.

Problems

∗ **10.13** Consider a shear deformation of a slab of elastic material in the xz-plane by a force in the x-direction. Assume that the sides of the slab are kept free to move, so that the only non-vanishing components of the strain tensor are $u_{xy} = u_{yx} = \alpha/2$. Show that the displacement becomes

$$u_x = \alpha y \ , \tag{10.41}$$

$$u_y = -\left(1 - \sqrt{1-\alpha^2}\right) y \ . \tag{10.42}$$

for a deformation which is not assumed to be small. Describe what happens for $\alpha \to 1$.

Answers

10.13 Assuming that only $u_x = \alpha y$ we get from (10.44)

$$2u_{xx} = (\nabla_x u_y)^2 = 0 \ , \tag{10.43a}$$

$$2u_{yy} = 2\nabla_y u_y + \alpha^2 + (\nabla_y u_y)^2 = 0 \ , \tag{10.43b}$$

$$2u_{xy} = \nabla_x u_y + \alpha + \nabla_x u_y \nabla_y u_y = \alpha \ . \tag{10.43c}$$

The first tells us that u_y only depends on y and the second that $\nabla_y u_y = -1 + \sqrt{1-\alpha^2}$. For $\alpha \to 1$ we find $u_y \to -y$ or $y' = y + u_y \to 0$, showing that the material has collapsed like a picket fence.