Nuclear Physics B73 (1974) 523–535. North-Holland Publishing Company

RADIATIVE DECAY OF AN EXCITED NEUTRINO IN GAUGE MODELS *

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Received 31 December 1973

Abstract: We study the decay of a heavy neutral lepton into a neutrino and a photon. In theories with a charged intermediate vector boson the decay rate is finite (in the one-loop approximation), provided the anomalous magnetic moment of the vector boson is exactly one, as it is in unified gauge models of weak and electromagnetic interactions. Our results exhibit strong model dependence.

1. Introduction

In gauge models of weak and electromagnetic interactions the otherwise bad high-energy behaviour is cancelled by introducing new fundamental particle exchanges. Depending on the model the new particles may be various combinations of heavy vector bosons, heavy leptons, and scalars ‡. In the Georgi-Glashow model [4] a neutral heavy lepton, E^o, is introduced along with a charged heavy lepton E^+ and a Higgs scalar. In the second model of Prentki and Zumino [5] the same particles appear together with a heavy neutral boson, Z^o. Other models involving an E^o have been discussed by Bjorken and Llewellyn Smith [3]. The E^o has exactly the same quantum numbers as the electron neutrino, ν_e, except for the mass, and may thus appropriately be called an "excited" neutrino. Such particles have been discussed before the advent of gauge theories [6] ‡‡. We shall attempt in this paper to keep

* This work was initiated at the Workshop on Weak Interactions at Billingehus, Skövde, Sweden, July 1973, and is supported in part by the Fonds zur Förderung der wissenschaftlichen Forschung in Österreich, Projekt Nr. 1905.

** Supported in part by Kulturamt der Stadt Wien.

‡ For a review of the unified models see for example Lee [1] or Llewellyn Smith [2] or Pietschmann [19]. Bjorken and Llewellyn Smith [3] give a detailed recipé for the construction of arbitrary gauge models, and discuss the decays of the new particles.

‡‡ There is even an unconfirmed claim that the excited muon neutrino has been seen [7].

the discussion general and not specialize to any specific model before it becomes unavoidable.

The main decay modes of a neutral excited neutrino may be classified as purely weak, simulating μ-decay

$$E^o \to e^- \nu_e e^+ , \tag{1.1}$$

$$E^o \to e^- \nu_\mu \mu^+ , \tag{1.2}$$

hadronic, simulating inverse π-decay,

$$E^o \to e^- \pi^+ , \tag{1.3}$$

$$E^o \to e^- \pi^+ \pi^o , \tag{1.4}$$

and electromagnetic

$$E^o \to \nu_e \gamma . \tag{1.5}$$

We have here only listed what we believe to be the dominant modes. Further hadronic modes may easily be invented, provided the E^o mass is sufficiently large. Similarly if other sequential or excited heavy leptons exist with mass below the E^o mass there will be further weak or electromagnetic channels open. In writing down the decay modes we have left out the ones entirely due to neutral currents different from the electromagnetic current. Examples of such processes are

$$E^o \to \nu_e \bar{\nu}_e \nu_e , \tag{1.6}$$

$$E^o \to \nu_e \bar{\nu}_\mu \nu_\mu , \tag{1.7}$$

$$E^o \to \nu_e \pi^o . \tag{1.8}$$

We remark that neutral currents responsible for these decays also could give contributions to (1.1).

In this paper we shall be concerned with the radiative decay (1.5). Its amplitude must be zero in the Born (tree) approximation in all theories with minimal electromagnetic couplings to the leptons, i.e. the first contribution to (1.5) must contain one loop. Bjorken and Llewellyn Smith [3] make the educated guess

$$\frac{\Gamma(E^o \to \nu_e \gamma)}{\Gamma(E^o \to e^- \nu_\mu \mu^+) + \Gamma(E^o \to e^- \nu_e e^+)} \sim 6 \frac{\alpha}{\pi} , \tag{1.9}$$

but warn that it may be wrong by an order of magnitude or more.

The initial motivation for the present calculation was partly to make yet another verification of the renormalizability of the gauge models by exposing the cancellation of divergences, and partly to improve the reliability of the value for the branching ratio (1.9). We shall see, however, that the decay rate is finite in all theories in which the charged intermediate vector boson, W, has anomalous magnetic moment exactly equal to 1, i.e. where the g-factor is equal to 2. This is a consequence of the Yang-Mills structure which is common to all unified gauge models and this calculation is therefore not very interesting from the point of view of the renormalizability of these theories. A similar situation occurs in the calculation of the weak contribution to the $g-2$ of the muon [8–10] and the quadrupole moment of the W itself [8, 11]. Both of these quantities are finite in lowest order when $g_W = 2$.

The paper is organized in the following way. In sect. 2 we discuss the kinematics and write down the general Lagrangian. In sect. 3 we show some details of the calculation and in sect. 4 we discuss the results. Sect. 3 may be skipped without loss of continuity.

2. Kinematics and the effective Lagrangian

In the following we use the notation indicated in fig. 1, where p is the momentum of the E^o of mass M, q the momentum of the neutrino and k the momentum of the photon *. The most general amplitude compatible with Lorentz invariance, current conservation and left-handedness of the neutrino is of the form

$$T = \bar{u}_\nu (1+\gamma_5) \lambda \frac{e}{M} \epsilon^\mu \, i\sigma_{\mu\nu} k^\nu u_E \, . \tag{2.1}$$

Here $\bar{u}_\nu$ is the neutrino spinor, $1+\gamma_5$ secures that it is left-handed **, λ is dimensionless and controls the magnitude of the matrix element, e/M is the transition magneton, ϵ^μ the photon polarization vector, $i\sigma_{\mu\nu} k^{\bar{\nu}}$ the magnetic moment current and u_E the spinor of the excited neutrino.

The unpolarized decay rate is then simply

$$\Gamma(E^o \to \nu\gamma) = M\alpha |\lambda|^2 \, , \tag{2.2}$$

where $\alpha = e^2/4\pi = \frac{1}{137}$ is the fine-structure constant.

As mentioned in the introduction the finiteness of the amplitude in lowest order is not dependent on the peculiarities of gauge theories except for the fact that the W anomalous magnetic moment is equal to 1. We shall therefore basically use the

* We use the metric with signature (+ - - -). Thus the kinematical invariants are $p \cdot k = q \cdot k = p \cdot q = \frac{1}{2} M^2$.

** We use γ-matrices satisfying $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g_{\mu\nu}$, and we take $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, and $\sigma_{\mu\nu} = \frac{1}{2} i [\gamma_\mu, \gamma_\nu]$. See Pietschmann [20].

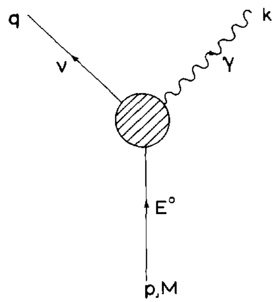

Fig. 1. Amplitude for the decay $E^0 \to \nu\gamma$ with momentum assignments indicated.

Lagrangian of old-fashioned (unrenormalizable) intermediate vector boson theory, which is of the form

$$\begin{aligned}\mathcal{L} &= \bar{\nu} i \partial\!\!\!/ \nu + \bar{E}^0 (i\partial\!\!\!/ - M) E^0 + \bar{e}(i\partial\!\!\!/ + eA\!\!\!/ - m)e - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ &- \tfrac{1}{2} \overline{W}_{\mu\nu} W^{\mu\nu} + \mu^2 \overline{W}_{\mu} W^{\mu} + ieKF^{\mu\nu} \overline{W}_{\mu} W_{\nu} \\ &+ g\bar{e}\gamma_{\mu}(1-\gamma_5)\nu \overline{W}^{\mu} + \text{h.c.} \\ &+ g\bar{e}\gamma_{\mu}(g_V + g_A\gamma_5) E^0 \overline{W}^{\mu} + \text{h.c.}\end{aligned} \tag{2.3}$$

Here ν is the neutrino field, E^0 the excited neutrino field, e the electron field, A_{μ} the Maxwell-field with field strengths $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$, and W_{μ} the positively charged intermediate vector boson field with field strengths $W_{\mu\nu} = (\partial_{\mu} + ieA_{\mu}) W_{\nu} - (\partial_{\nu} + ieA_{\nu}) W_{\mu}$. We have denoted the electron mass by m and the W-mass by μ. This Lagrangian is constructed from the Lagrangian of the uninteracting fields by coupling the charged particles minimally to the electromagnetic field and afterwards adding in a non-minimal anomalous magnetic moment term (last term in second line), an ordinary weak interaction term (third line) and finally an arbitrary (V,A) coupling between e, E^0 and W.

Our Lagrangian is model-independent in the sense that all renormalizable unified gauge theories of weak and electromagnetic interactions (including an excited neutrino E^0) must contain it as a part, for suitable choices of the coupling constants g_V and g_A (of order 1). The coupling constant g is always fixed by the Fermi constant

$$\sqrt{2}\,\frac{g^2}{\mu^2} = G = 10^{-5} M_p^{-2} , \tag{2.4}$$

and the anomalous magnetic moment K is equal to 1 in these theories. We shall return to the discussion of the special models in sect. 4.

In third order of approximation the Lagrangian (2.3) gives rise to the two diagrams

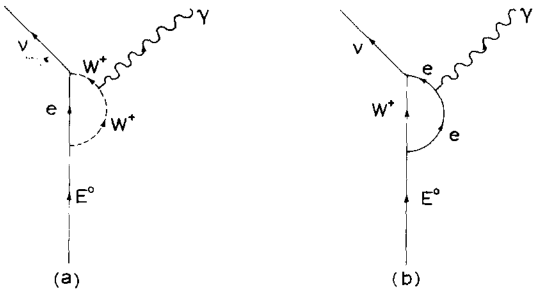

Fig. 2. Amplitude for $E^0 \to \nu\gamma$ in the one-loop approximation.

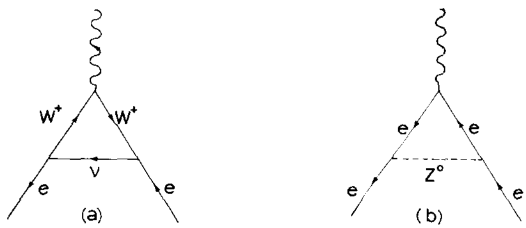

Fig. 3. W and Z^0 contribution to the electron's electromagnetic vertex.

shown in fig. 2, for which the amplitude is of order g^2e. As we shall see in the following section these diagrams give separately finite contributions to λ. Qualitatively this may be understood in the following way. Diagrams 2a and 2b are structurally identical to the weak corrections to the electromagnetic vertex of the electron shown in fig. 3. In particular the transition magnetic moments in which we are interested, correspond to the weak corrections to the magnetic moment of the electron from the diagrams in fig. 3. It is well-known that these are finite for $K = 1$ *.

3. Evaluation of the amplitudes

The two diagrams in fig. 2 have a number of common factors that we isolate by writing

$$T = eg^2 \epsilon^\mu \bar{u}_\nu (1+\gamma_5) S_\mu (g_V + g_A \gamma_5) u_{E^0} , \tag{3.1}$$

where (corresponding to the two diagrams)

* The anomalous magnetic moment from fig. 3a was first (correctly) evaluated by Brodsky and Sullivan [9] and Burnett and Levine [10] for arbitrary K using a "ξ" cut-off of the vector propagator. The contribution from fig. 3b has been evaluated by a number of authors. We refer to the general reviews of the subject [12–14].

$$S_\mu = S_\mu^{\mathrm{a}} + S_\mu^{\mathrm{b}} , \tag{3.2}$$

with

$$S_\mu^{\mathrm{a}} = i \int \frac{\mathrm{d}^n l}{(2\pi)^n} \gamma_\rho (\not{l} - m)^{-1} \gamma_\sigma (D(l_2) W_\mu (l_2 l_1) D(l_1))^{\rho\sigma} , \tag{3.3}$$

$$S_\mu^{\mathrm{b}} = -i \int \frac{\mathrm{d}^n l}{(2\pi)^n} \gamma_\rho (\not{l}_2 - m)^{-1} \gamma_\mu (\not{l}_1 - m)^{-1} \gamma_\sigma D^{\rho\sigma}(l) . \tag{3.4}$$

In both cases we have

$$l_1 = p - l , \tag{3.5}$$

$$l_2 = q - l . \tag{3.6}$$

In eq. (3.3) we use a matrix notation in order to avoid too many vector indices. The vector propagator

$$D^{\rho\sigma}(l) = \left(\frac{1 - l\, l/\mu^2}{l^2 - \mu^2} \right)^{\rho\sigma} = \frac{g^{\rho\sigma} - l^\rho l^\sigma/\mu^2}{l^2 - \mu^2} \tag{3.7}$$

is in its unitary form, and the W electromagnetic vertex $W_\mu^{\rho\sigma}(l_2 l_1)$ has the form (for $K = 1$)

$$W_\mu (l_2 l_1) = (l_2 + l_1)_\mu + e_\mu (l_1 - 2l_2) + (l_2 - 2l_1) e_\mu . \tag{3.8}$$

The symbols e_μ are the unit vectors, i.e. $(l e_\mu)^{\rho\sigma} = l^\rho \delta^\sigma_\mu$.

Power counting indicates that (3.3) is superficially quartically divergent while (3.4) is quadratically divergent. The reduction of divergences only shows up after a certain amount of algebraic manipulation and transformation of the integrals has been performed. It is therefore necessary to regulate the naive expressions and we have chosen to use the t'Hooft-Veltman dimensional regularization scheme [15] because of its calculational efficiency.

The opposite sign in (3.3) and (3.4) is essentially due to the fact that W^+ and e^- have opposite charges. One may check the relative sign by observing that S_μ structurally is related to the neutrino vertex correction shown in fig. 4. As it is known [11] that the neutrino self-charge vanishes we expect that $S_\mu = 0$ for $k_\mu = 0$. In fact putting $q = p$ in (3.3) and (3.4) and using the identities

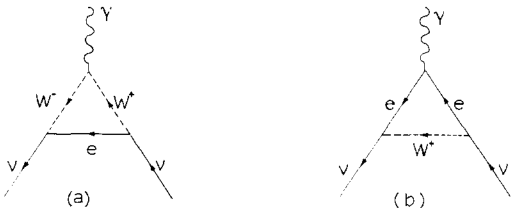

Fig. 4. Neutrino vertex correction.

$$\frac{\partial}{\partial l^\mu}\frac{1}{\not{l}-m}=-\frac{1}{\not{l}-m}\gamma_\mu\frac{1}{\not{l}-m}\,, \tag{3.9}$$

$$\frac{\partial}{\partial l^\mu}D(l)=-D(l)W_\mu(l,l)D(l)\,, \tag{3.10}$$

we find that we may write S_μ as the integral over a total derivative

$$S_\mu=i\int\frac{\mathrm{d}^n l}{(2\pi)^n}\frac{\partial}{\partial l^\mu}\left(\gamma_\rho(\not{l}-m)^{-1}\gamma_\sigma D^{\rho\sigma}(p-l)\right), \tag{3.11}$$

which indeed vanishes when the theory is dimensionally regularized [15] *.

It is very easy to generate a large number of terms in evaluating S_μ^a. It is therefore imperative to use a number of simplifying tricks before introducing Feynman parameters. The first of these is to use the Gordon transformation in eq. (2.1). It takes the form

$$i\sigma_{\mu\nu}k^\nu\simeq p_\mu+q_\mu-M\gamma_\mu\,, \tag{3.12}$$

where the sign $\simeq$ indicates equality when sandwiched between the relevant spinors. Thus the terms that we are interested in may be found as coefficients of $(p+q)_\mu$ while terms proportional to γ_μ may be dropped. Terms proportional to $k_\mu=(p-q)_\mu$ may be dropped because of gauge invariance. Likewise terms containing $\not{q}$ may be reduced by moving $\not{q}$ to the left where it eventually hits the neutrino spinor and vanishes. Terms containing $\not{p}$ may be moved to the right until they hit the E^0 spinor where $\not{p}=M$.

The vector part of the integrand of S_μ^a may be written in the following way

$$D(l_2)W_\mu(l_2l_1)D(l_1)\simeq\frac{(l_2+l_1)_\mu(1-l_2l_1/\mu^2)+2(e_\mu k-ke_\mu)}{D_2D_1}+\frac{l_2e_\mu}{\mu^2D_2}+\frac{e_\mu l_1}{\mu^2D_1}\,, \tag{3.13}$$

* Perhaps we should remark that in this calculation the particles are assumed to be off the mass shell so that it makes sense to put $q=p$, and that the vanishing of S_μ does not depend on K being equal to 1.

where $D_i = l_i^2 - \mu^2$. Observe that the most divergent part has dropped out because $l_2 \cdot W_\mu \cdot l_1 \simeq 0$. The last two terms cannot give any contribution because they lead to expressions of the form $F(\not{q})\gamma_\mu \sim \gamma_\mu$ or $\gamma_\mu F(\not{p}) \sim \gamma_\mu$ after integration; i.e. they may be dropped. The remaining most divergent terms are the ones containing $(l_2+l_1)_\mu l_2 l_1$. But writing

$$\not{l}_2(\not{l}-m)^{-1}\not{l}_1 \simeq -\not{l}_1 + m - m(\not{l}-m)^{-1}(\not{p}-m)\,,$$

we see that we may drop the first two of these because the expression

$$\int \mathrm{d}^n l(\not{l}_1 - m)(l_1+l_2)_\mu / D_1 D_2$$

can only depend on k_μ. Hence we arrive at the expression

$$S_\mu^{\mathrm{a}} \simeq i\int \frac{\mathrm{d}^n l}{(2\pi)^n}\, \frac{1}{D_1 D_2 D_3}\, \{(l_1+l_2)_\mu\, \gamma_\rho(\not{l}+m)\gamma^\rho$$

$$+ (l_1+l_2)_\mu \frac{m}{\mu^2}(\not{l}+m)(\not{p}-m) + 2\gamma_\mu(\not{l}+m)\not{k} - 2\not{k}(\not{l}+m)\gamma_\mu\}\,, \tag{3.14}$$

where $D_3 = l^2 - m^2$. Similarly we find from (3.4)

$$S_\mu^{\mathrm{b}} = -i\int \frac{\mathrm{d}^n l}{(2\pi)^n}\, \frac{1}{D_1' D_2' D_3'}\, \{\gamma_\rho(\not{l}_2+m)\gamma_\mu(\not{l}_1+m)\gamma^\rho$$

$$+ \frac{m}{\mu^2}(\not{l}_2+m)\gamma_\mu(\not{l}_1+m)(\not{p}-m)\}\,, \tag{3.15}$$

where D_i' are obtained from D_i by interchanging m and μ.

There is still a superficial logarithmic divergence but it is easily seen that it can only occur in terms $\sim \gamma_\mu$ and will be dropped. Thus we have demonstrated the finiteness of S_μ.

The remainder of the calculation is now straightforward. If one first combines the denominators D_1 and D_2 in S_μ^{a} by means of the Feynman parameter y and afterwards combines the result with D_3 using x, one arrives at the following characteristic denominator after carrying out the momentum integrations

$$L = x\mu^2 + (1-x)m^2 - yx(1-x)M^2\,. \tag{3.16}$$

The denominator of S_μ^{b} is obtained by interchanging μ and m, an operation that may be counteracted by letting $x \to 1-x$. Hence S_μ^{a} and S_μ^{b} have the same denominator and may be added together. The result is

$$S_\mu = \frac{(p+q)_\mu}{32\pi^2}\,\frac{M}{\mu^2}\left(I_+ + I_-\frac{\not p}{M}\right), \tag{3.17}$$

with

$$I_+ = \frac{2m}{M}\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\ \frac{(1-x)(xyM^2-m^2)-4\mu^2 x}{L}, \tag{3.18}$$

$$I_- = \int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y\ \frac{4x(1-y(1-x))\mu^2 + 2(1-x)(1-xy)m^2}{L}. \tag{3.19}$$

These expressions are of course real below the threshold for the intermediate state, i.e. for $M < \mu + m$.

In the calculation of the integrals in diagram 2a one meets the same difficulties as in the calculation of the weak correction to the anomalous magnetic moment of the muon [11, 16, 17]. These so-called ambiguities in routing the integration momentum which give rise to finite anomalous contributions to the integrals are resolved in the renormalizable gauge models, where calculations can be carried out in manifestly renormalizable gauges [18] or by using the position space regularization procedure discussed by Kummer and Lane [21]. In our case the ambiguity has been settled by choosing a γ_5 that anticommutes with all γ-matrices in the derivation of eq. (3.1). In the dimensional regularization scheme no definition of γ_5 exists, which would preserve all Ward identities for $n \neq 4$ *. t'Hooft and Veltman [15] suggest a γ_5 that anticommutes with the "first four" components of γ_μ and commutes with the rest. It was pointed out in ref. [11], however, that such a choice does not lead to the correct value for the muon's anomalous magnetic moment. Choosing a γ_5 which anticommutes with all components of γ_μ leads in the case of the $g-2$ of the muon to the correct value [11] and we have therefore adopted this procedure.

4. Discussion of the results

The dimensionless parameter λ introduced in eq. (2.1) can now be determined

$$\lambda = \frac{GM^2}{32\pi^2\sqrt{2}}((g_\mathrm{V}+g_\mathrm{A})I_+ + (g_\mathrm{V}-g_\mathrm{A})I_-)\,. \tag{4.1}$$

The order of magnitude of the decay rate is thus

$$M\alpha\left(\frac{GM^2}{32\pi^2\sqrt{2}}\right)^2 = 3.4\times 10^{-9}\left(\frac{M}{M_p}\right)^5\ \mathrm{eV}\,. \tag{4.2}$$

* We would like to thank Professor W. Kummer for a discussion about this point.

In order to evaluate the integrals $I_\pm$ we assume that the charged vector boson is much heavier than any lepton, i.e. $\mu \gg M, m$. In this approximation we get from (3.18) and (3.19)

$$I_+ = -8\,\frac{m}{M} + \dots\,, \tag{4.3}$$

$$I_- = 3 - 3\,\frac{m^2}{\mu^2} + \frac{4}{3}\,\frac{M^2}{\mu^2} + \dots\,. \tag{4.4}$$

Hence unless $g_V = g_A$ we find in the simple model used here where $m \ll M$ that

$$\Gamma(E^o \to \nu\gamma) = \frac{9\,\alpha M(GM^2)^2}{2048\,\pi^4}\,(g_V - g_A)^2\,. \tag{4.5}$$

In the same model (in fact in all models) we have

$$\Gamma(E^o \to e^-\nu_e e^+) = \frac{M(GM^2)^2}{192\,\pi^3}\,\tfrac{1}{2}(g_V^2 + g_A^2)\,, \tag{4.6}$$

so that

$$\frac{\Gamma(E^o \to \nu\gamma)}{\Gamma(E^o \to e^-\nu_e e^+)} = \frac{27}{16}\,\frac{\alpha}{\pi}\,\frac{(g_V - g_A)^2}{g_V^2 + g_A^2}\,. \tag{4.7}$$

As the muon and electron (for $M_{E^o} \gg m_\mu$) give the same leading contribution we infer

$$\frac{\Gamma(E^o \to \nu\gamma)}{\Gamma(E^o \to e^-\nu_e e^+) + \Gamma(E^o \to e^-\nu_\mu \mu^+)} = \frac{27}{32}\,\frac{\alpha}{\pi}\,\frac{(g_V - g_A)^2}{g_V^2 + g_A^2}\,. \tag{4.8}$$

The right-hand side is maximally $\frac{27}{16}\alpha/\pi = 1.7\,\alpha/\pi$ which should be compared with the estimate (1.9), $6\,\alpha/\pi$, of Bjorken and Llewellyn Smith [3].

We now turn to the discussion of the various models. In the "2–2" model [3] only the right-handed part of E^o couples to the electron, i.e. $g_V = g_A = 1$. The leading contribution comes now from I_+ rather than I_- and we find

$$\frac{\Gamma(E^o \to \nu\gamma)}{\Gamma(E^o \to e^-\nu_e e^+) + \Gamma(E^o \to e^-\nu_\mu \mu^+)} = 12\,\frac{\alpha}{\pi}\left(\frac{m_e}{M_{E^o}}\right)^2\,, \tag{4.9}$$

which is very small due to the appearance of the electron mass.

In the second model of Prentki and Zumino [5] as well as in the Georgi-Glashow model [4] a heavy positively charged electron type lepton E^+ appears. It gives an amplitude described by the diagrams in fig. 5. They are of exactly the same structure as in fig. 2 and may be obtained from the same formulas, (3.18), (3.19) and (4.1), by making the appropriate substitutions, i.e.

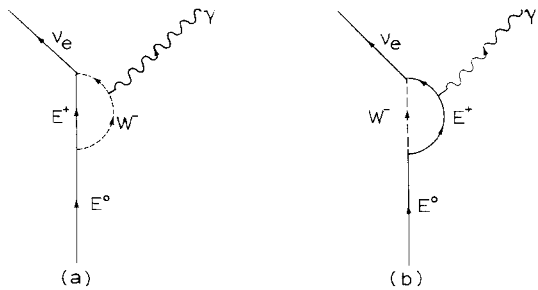

Fig. 5. Additional amplitude for $E^o \to \nu\gamma$ in models with an E^+.

$$\lambda = \frac{GM^2}{32\pi^2\sqrt{2}} \left((g_V+g_A)I_+ + (g_V-g_A)I_- - (g'_V+g'_A)I'_+ - (g'_V-g'_A)I'_-\right), \qquad (4.10)$$

where g'_V, g'_A are the $\bar{E}^oE^+W^-$ coupling constants defined as in (2.3). $I'_\pm$ are found by replacing m by M_{E^+}. The overall sign change is due to the change of sign of the charge of the particles emitting the photon. In the second model of Prentki and Zumino [5] one has $g_V = -g_A = -g'_V = g'_A = -1$ so that the contributions from fig. 2 and fig. 5 interfere constructively. Hence in the same approximation as before, ($\mu_W \gg M_{E^o}, M_{E^+}, m_e$), we get

$$\frac{\Gamma(E^o \to \nu\gamma)}{\Gamma(E^o \to e^-\nu_e e^+) + \Gamma(E^o \to e^-\nu_\mu\mu^+)} = \frac{27}{4}\frac{\alpha}{\pi}, \qquad (4.11)$$

which is almost exactly the result guessed by Bjorken and Llewellyn Smith [3] (eq. (1.9)).

In the Georgi-Glashow model [4] one finds $g_V = g'_V = \cot\frac{1}{2}\beta$ and $g_A = g'_A = \mathrm{tg}\,\frac{1}{2}\beta$ where β is the mixing angle. The contributions interfere destructively in this case but this is counteracted by the fact that the E^+ may have a mass comparable to E^o. The leading term in (4.4) is cancelled and we find from (4.3) *:

$$\frac{\Gamma(E^o \to \nu\gamma)}{\Gamma(E^o \to e^-\nu_e e^+) + \Gamma(E^o \to e^-\nu_\mu\mu^+)} = 48\,\frac{\alpha}{\pi}\,\frac{\cos^2\beta}{1+\cos^2\beta}\,. \qquad (4.12)$$

The right-hand side has a maximum $24\alpha/\pi = 5.5\%$ which is attained for $\beta = 0$. This value is, however, forbidden by the $g-2$ of the muon which disagrees with experiment unless $\sin\beta \gtrsim \frac{1}{3}$ [12]. This constraint is, however, not very serious. Taking it into account the maximum is reduced to 5.25%.

* We have here used the relation $M_{E^+}/M_{E^o} = 2\cos\beta$. The W-mass is $\mu_W = 53$ GeV $\sin\beta$ in this model. Thus in order that our approximation $\mu_W \gg M_{E^o}, M_{E^+}, m_e$ be valid we must require that β is not too close to 0° or 90°.

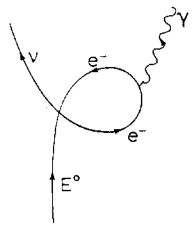

Fig. 6. Amplitude in 4-fermion theory.

Finally we should perhaps mention that if one attempts to calculate the decay rate in old-fashioned 4-fermion theory (fig. 6) the result is entirely dependent on the regularization procedure *.

5. Conclusions

We have evaluated the branching ratio of $E^0 \to \nu_e \gamma$ in several different models of weak interactions ranging from old-fashioned four-fermion theory over pre-gauge W theory, to the modern renormalizable unified theories of weak and electromagnetic interactions. The branching ratio is extremely model dependent as witnessed by (4.8), (4.9), (4.11) and (4.12). The numerical values range from zero to 5% of the leptonic decay modes, ($E^0 \to e^- \nu_e e^+, e^- \nu_\mu \mu^+$). The calculation (sect. 3) has been organized in such a way that it is easy to evaluate the branching ratio in any model not considered here.

The authors would like to thank the organizers of the "Workshop on Weak Interactions" held in Billingehus, Skövde, Sweden in July 1973 for their kind hospitality. We would also like to thank the participants for many stimulating discussions and comments.

Note added in proof

Our result for the Georgi-Glashow model (eq. (4.12)) agrees with the recent calculation of Pi and Smith [22].

* If one regulates the diagram of fig. 6 using a Pauli-Villars procedure the result is identically zero ($I_+ = I_- = 0$). If one uses dimensional regularization the result is non-zero due to the fact that $\gamma_\mu \gamma^\mu = n$ and not 4. That changes the γ-algebra in the numerator and leaves a term proportional to $n-4$. This zero is however removed by a pole, $1/(n-4)$, corresponding to the logarithmic divergence of the momentum integral and the result is finite ($I_+ = -4\, m/M, I_- = 0$). If the value of the diagram in fig. 6 is defined to be the leading term in fig. 2b for $\mu \to \infty$ (which is proportional to $1/\mu^2$) a finite value again results ($I_+ = -4\, m/M, I_- = \frac{4}{3}$).

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