TWO-BARYON APPROXIMATION TO THE DECAY OF NEUTRAL PSEUDOSCALAR MESONS

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The decay widths of η and π^0 are calculated using two intermediate baryons and SU(3) coupling constants. Within reasonable limits one gets agreement with the latest experimental value for the π^0 decay width. The total η width is predicted to be 1.09 keV.

The 2γ-decay of the pseudoscalar mesons is usually calculated using vector meson intermediate states [1], e.g. by use of diagrams of the type shown in fig. 2(e). Such a calculation requires that the ρ-π-γ coupling constant is of the order e. However, recently Donnachie and Shaw [2] have shown that the ρ-π-γ coupling constant comes out consistent with zero (from an analysis of photo-production). It therefore appears to be impossible to assume that the vector meson intermediate states give rise to the leading contributions to the 2γ-decay.

We therefore propose that the 2γ-decays of π^0 and η are dominated by two-baryon intermediate states. The diagram for this process is shown in fig. 1, where the neutral meson decays into two photons via an intermediate baryon-antibaryon state. The invariant amplitude corresponding to fig. 1 is given by the expression

$$\langle k_1 k_2 | T_i | p\rangle = 2\mathrm{i} g_i e^2 \int \mathrm{d}q/(2\pi)^4 \times \times \mathrm{Tr}\left[\gamma_5 \frac{1}{\not q - \not k_1 - M_i} \not e_1 \frac{1}{\not q - M_i} \not e_2 \frac{1}{\not q + \not k_2 - M_i}\right], \tag{1}$$

where M_i is the mass of the baryon B_i, g_i is the meson-baryon coupling constant and the meaning of the other symbols can be inferred from fig. 1. The factor of 2 comes in because there are actually two diagrams with the same amplitude. It has been known for a long time that this expression comes out finite [3]. Using standard techniques one obtains

$$\langle k_1 k_2 | T_i | p\rangle = -8\mathrm{i} M_i g_i e^2 [e_1 e_2 k_1 k_2] I_i \ , \tag{2}$$

where

$$[abcd] = \epsilon_{\mu\nu\rho\sigma}\, a^\mu b^\nu c^\rho d^\sigma \ , \tag{3}$$

and

$$I_i = \int \frac{\mathrm{d}q}{(2\pi)^4} \frac{1}{[(q-k_1)^2 - M_i^2][q^2 - M_i^2][(q+k_2)^2 - M_i^2]}$$

$$= \frac{1}{16\pi^2 \mathrm{i} M_i^2} \int_0^1 \mathrm{d}\alpha \int_0^{1-\alpha} \mathrm{d}\beta \frac{1}{1 - \alpha\beta\xi_i^2}$$

$$= \frac{1}{32\pi^2 \mathrm{i} M_i^2} \left(\frac{2\arcsin(\frac{1}{2}\xi_i)}{\xi_i}\right)^2 \ . \tag{4}$$

Here

$$\xi_i = m/M_i \ ,$$

and m is the mass of the neutral meson. In order to use SU(3) we take for M_i the mean mass $M =$ = 1154 MeV of the baryon octet and the SU(3) coupling constants g_i. Summing (2) over all charged baryons and using standard techniques, we find the following expression for

$$\Gamma_{2\gamma} = m\left(\frac{\alpha}{\pi}\right)^2 \frac{(\sum_i g_i)^2}{4\pi} \frac{\xi^2}{16} \left(\frac{2\arcsin(\frac{1}{2}\xi)}{\xi}\right)^4 , \tag{5}$$

$$\left(\frac{2\arcsin(\frac{1}{2}\xi)}{\xi}\right)^4 = 1 + \tfrac{1}{6}\xi^2 + \tfrac{7}{240}\xi^4 + \dots \tag{5a}$$

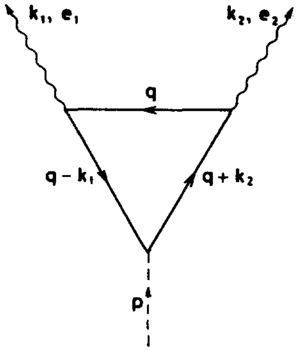

Fig. 1.

The sum over the various coupling constants in eq. (5) can be performed using SU(3) coupling constants. One finds for η

$$\sum_i g_i = (2/\surd 3)\, \alpha_S g\ , \tag{6}$$

and for π^0

$$\sum_i g_i = 2\alpha_S g\ . \tag{7}$$

Here g is the strong interaction coupling constant ($g^2/4\pi = 14.8$) and α_S is the strong interaction SU(3) mixing parameter, defined by

$$\langle B_i | j_k(0) | B_\ell \rangle = -ig\bar{u}_i \gamma_5 u_\ell [\alpha_S d_{ik\ell} + (1-\alpha_S) f_{ik\ell}]\ , \tag{8}$$

where $j_k(x)$ is the current of the meson octet. The strong mixing parameter α_S has been determined from semiphenomenological fits [4] and from PCAC [5]. The result is

$$\alpha_S = 0.73 \pm 0.03. \tag{9}$$

Inserting eqs. (6), (7) and (9) into eq. (5) one obtains

$$\Gamma(\eta \to 2\gamma) = 447 \text{ eV}\ , \tag{10}$$

$$\Gamma(\pi^0 \to 2\gamma) = 19.8 \text{ eV}\ . \tag{11}$$

From the experimental branching ratios the life times are found to be

$$\tau_\eta = (1.47 \pm 0.05) \times 10^{-18} \text{ sec}\ , \tag{12}$$

$$\tau_{\pi^0} = (0.34 \pm 0.03) \times 10^{-16} \text{ sec}\ . \tag{13}$$

The latest experimental value for τ_{π^0} is [6]

$$\tau_{\pi^0}\,(\text{exp}) = (0.74 \pm 0.11) \times 10^{-16} \text{ sec}\ . \tag{14}$$

It is seen that eq. (13) is not in perfect agreement with the experimental life time. The discrepancy is at least a factor 1.6. Thus, in the invariant amplitude $\langle k_1 k_2 | T | p \rangle$ there lacks a

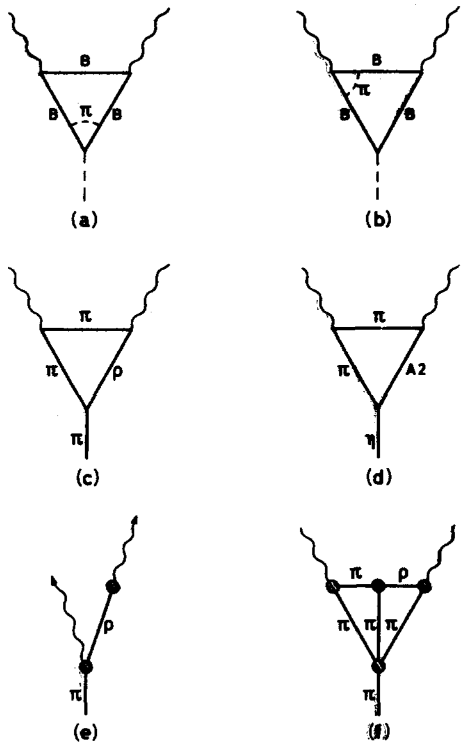

Fig. 2.

factor $1/\surd 1.6 \approx 0.8$. Therefore, the two-baryon approximation to the invariant amplitude $\langle k_1 k_2 | T | p \rangle$ is not the only contribution to the 2γ-decay. On the other hand, the two-baryon approximation accounts for the 2γ-decay within 20%, and we can therefore conclude that it is very likely that *the diagram in fig. 1 gives the main contribution to the 2γ-decay.*

Finally, we want to give arguments why the two-baryon approximation in fig. 1 gives the main contribution to the decay of the neutral pseudoscalar mesons. The main question is:

Why do we only take into account the diagram shown in fig. 1, and not graphs of the types shown in fig. 2? Diagrams of the types (a) and (b) in fig. 2 give corrections to the various vertices involved in the primary graph of fig. 1. On account of the convergence of the primary graph, it may be expected that everything is almost put on the mass shell by the propagator singularities. The diagrams of type (a) will mostly contribute to the renormalization of the meson-nucleon coupling constant. Diagrams of type (b) give rise to form factors of the baryons, but since the baryons are assumed to be approximately on the mass shell, the charge distribution terms will mostly contribute to the renormalization of the charge of the baryons, and the magnetic terms will presumably be very small. Propagator corrections may be treated in the same way.

The reason why we have omitted the diagram (c) is (as mentioned before) that Donnachie and Shaw [2] have shown that the ρ-π-γ coupling constant comes out (from photo-production) consistent with zero. A quite similar remark applies to the A2-η-π vertex, since the branching ratio of A2 $\rightarrow \eta + \pi$ is only 2%.

Diagrams with K and K* are also of type (c). But presumably the K-K*-γ coupling is of the same order of magnitude as the ρ-π-γ coupling. Hence, such diagrams may also be omitted.

Diagrams of type (f) are too complicated to be evaluated. Furthermore, the diagram (f) is to a large extent given by the diagram (c), and since (c) was negligible it is tempting to expect that (f) is also negligible.

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ADLER-WEISBERGER SUM RULES FOR HIGHER SPIN PARTICLES*

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It is pointed out that, for particles of spin J or $J+\frac{1}{2}$ (J an integer), there are $J+1$ independent Adler-Weisberger sum rules. The case of $\pi\rho$ scattering is treated as an example to show that more useful information can be obtained by considering all of them.

The commutation relations of the chiral SU(3) $\times$ SU(3) algebra [1] were used by Adler and Weisberger [2] to derive a sum rule expressing the axial current renormalization constant in terms of the forward scattering amplitude for pions on nucleons. The great success of the sum rule has encouraged attempts to apply the technique to particles other than nucleons. Since total cross sections for pions on particles other than nucleons are not known, it has generally been assumed that these cross sections are dominated by contributions from a few resonant states. The sum rule then yields information about coupling constants [3].

It is the purpose of this note to point out that 1. for a particle of spin J (or a fermion of spin $J+\frac{1}{2}$) ** there are $(J+1)$ sum rules corresponding to the $(J+1)$ independent forward scattering amplitudes in a helicity representation; 2. these sum rules are not equivalent since only intermediate states of total angular momentum $\geqslant \lambda$ can contribute to the sum rule for a particular helicity λ;

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** We restrict our attention to bosons since most work in this area has been concentrated on meson couplings.